**Barron’s Let’s Review Regents – Algebra II**

# Chapter 1: Polynomial Expressions and Equations

## Polynomial Arithmetic

**Multiplying a Polynomial by a Constant**

**Adding Polynomials**

1. Combine the two terms.
2. Combine the two terms.
3. Combine the two constant terms.

**Subtracting Polynomials**

The minus sign can be thought of as multiplying by a negative one. Distribute the multiplication through the second polynomial.

**Multiplying Polynomials**

**FOIL**

1. **F**irst terms in each expression.
2. **O**uter terms in each expression.
3. **I**nner terms in each expression.
4. **L**ast terms in each expression.

**Multiplication Patterns**

**Perfect Squares**

The coefficient of the -term will be twice the constant term.

**Difference of Perfect Squares**

**Dividing Polynomials**

Answer:

### Check Your Understanding of Section 1.1

1. Multiple-Choice
2. What is   
   **(1)**
3. What is ?  
   **(2)**
4. What is ?  
   **(4)**
5. What is ?  
   **(1)**
6. What is ?  
   **(4)**
7. What is ?  
   **(4)**
8. What is ?  
   **(1)**
9. What is ?  
   **(2)**
10. What is ?  
    **(1)**
11. What is ?  
    **(4)**
12. What is ?  
    **(3)**
13. *Show how you arrived at your answers*.
14. Simplify .
15. Zahra calculated   
     as  
     and got   
     What mistake did Zahra make?  
      
    **Zahra made the mistake of not distributing the minus sign in the second expression.**  
    **It should have been:**
16. If , what is the value of *a*?  
      
    **)** (  
     (ck)
17. If , what is the value of *a*?
18. Simplify .

## 1.2 Polynomial Factoring

*Factoring* an integer means finding two other integers (other than 1) whose product is equal to the original integer. The integer 15 has two factors: 3 and 5.

When a polynomial is factored, the factors can provide useful information about the polynomial that was not apparent in the non-factored form.

Just like some numbers can’t be factored (for example, 7, and other prime numbers), some polynomials cannot be factored either. When a polynomial can be factored, there are several different methods of obtaining the factorization, depending on the polynomial.

**Greatest Common Factor Factoring**

*Greatest common factor* factoring is the first type of factoring you should always try. If all the terms of a polynomial have a common factor, that common factor can be “factored out.” Often, the only common factor is 1, in which case this type of factoring is not useful.

**Factoring a Quadratic Trinomial into the Product of Two Binomials**

What is the opposite of FOIL? The opposite is factoring a quadratic trinomial like into the product of two binomials.

If the trinomial is of the form , find two numbers that have a sum of b and a product of c.

Perfect Square Trinomial Factoring

The trinomials , and   
, are three examples of perfect square trinomials. These can be factored into , , and , respectively.

The way to recognize a perfect square trinomial of the form is to compare *c* to .

If , the trinomial is a perfect square and can be factored as

**Difference of Perfect Squares Factoring**

A quadratic expression like is known as the difference between two perfect squares since each of the terms is a perfect square and there is a subtraction sign between the two terms.

**Factoring Cubic Expressions by Grouping**

Polynomial expressions that have one of the variables raised to the third power are called cubic polynomials. Generally, they are very difficult to factor. Sometimes a technique called factor by grouping can be used to factor certain cubic polynomials.

Factor by grouping when you have a four-term cubic expression and you factor a common factor from the first two terms and another common factor from the last two terms. Then you cross your fingers and hope there will be a new common factor that you can then factor out.

For example, the polynomial can be factored this way.

An can be factored out of the first two terms, and a 4 can be factored out of the last two terms.

At this point, notice that both of the terms have a factor of . This can be factored out to become  
.

**Factoring More Complicated Expressions**

Polynomials that have exponents greater than 3 can sometimes be factored by rewriting the polynomials in an equivalent form that can be factored with other methods.

The expression has an exponent of 6. Since 6 is an even number, can be expressed as . Since 16 is a perfect square, the original expression can be written as , which can then be factored with the difference of perfect squares pattern: .

**Algebra Identities**

When two algebraic expressions can be simplified to the same expression, it is called an identity. Proving that something is an identity requires simplifying one or both sides of the equation until the two sides are identical. Until an identity is proved, there will be a small question mark over the equal sign like ≟. After the identity is established, the question mark over the equals sign is replaced with a .

**Number Theory Proofs**

Sometimes a theorem about numbers can be proved by turning the theorem into an identity to be verified.

### Check Your Understanding of Section 1.2

1. Multiple-Choice
2. Which shows factored?  
   **(1)**
3. Which shows factored?  
   **(1)**
4. Which shows factored?  
   **(3)**
5. Which shows factored?  
   **(4)**
6. Which shows factored?  
   **(4)**
7. Which shows factored?  
   **(2)**
8. Which shows factored?  
   **(2)**
9. Which shows factored?  
   **(1)**
10. Which shows factored?  
    **(1)**
11. Which shows ?  
    **(3)**
12. Show how you arrived at your answers.
13. Factor .  
    Factors: 1, 3, 2x, x
14. Factor .  
    Factors: 17, 39
15. What value for *c* can be factored into ?  
    *c* = 81
16. How can the fact that be used to find the factors (not including (1 or 551) of 551?  
      
    **Difference of perfect squares**
17. Completely factor .  
    Factors: (1,36), (2,18), (3,12), (4,9), (6,6)

## 1.3 The Remainder Theorem and the Factor Theorem

When something is a factor of a number, like 5 is a factor of 10, there will be no remainder when the number is divided by the factor. This is called *the factor theorem*. When something is not a factor of a number, like 3 is not a factor of 10, there will be some remainder when the number is divided by the factor. With polynomial division, there is a theorem called *the remainder theorem* that enables you to determine the remainder of some divisions without going through the long division process.

**The Remainder Theorem**

If you divide the polynomial function   
 by with the long division process, you get remainder 5.The remainder theorem says that you will also get the number 5 if you substitute +4 (the opposite of -4 in the divisor) into the   
.

To check if the remainder theorem works for this example, evaluate

**Math Facts**

The remainder theorem says that the remainder when a polynomial equation is divided by is equivalent to the value of the polynomial when is substituted for . If the expression is , then substitute into the polynomial.

**The Factor Theorem**

When a binomial like is a factor of a polynomial like it means there will be remainder of zero when is divided by If you evaluate the polynomial   
 for , it becomes   
, just as the remainder theorem predicted. From this, we get the factor theorem.

**Math Facts**

The factor theorem says that if is a factor of a polynomial, then the value of the polynomial when is substituted for will be 0.

### Check Your Understanding of Section 1.3

1. Multiple-Choice
2. What is the remainder when is divided by ?  
   (3\*3\*3)+(3\*3)-(9\*3)+5 = 27+9-27+5 = 14
3. What is the remainder when   
    is divided by ?  
   **(2) 3**
4. If , what is the remainder when is divided by   
   **(2)**
5. If the remainder when is divided by is 7, what is the value of a?  
   **(4) 5**
6. If the remainder when is divided by is 97, what is the value of ?  
   **(3) 9**
7. If , what is the remainder when is divided by ?  
   **(4) 0**
8. Which of the following is a factor of   
   ?  
   1 => Remainder -30  
   2 => 8 + 12 – 20 – 24 = -24  
   3 => 27 + 27 – 30 -24 = 54 – 54 = 0  
   **(3)**
9. Which of the following is a factor of   
   ?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x |  |  |  |  |  | Total |
| 3 | 162 | -243 | -81 | 138 | 24 | 0 |
| -4 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 1 | 2 | -9 | -9 | 46 | 24 | 54 |

**(1)**

1. If is a factor of , what is the value of ?  
   **(4) -60**
2. If is a factor of , what is the value of ?  
   **(3) 6**
3. *Show how you arrived at your answers*.
4. What is the remainder when   
    is divided by ?  
   f(4)
5. . If , what is the reminder when is divided by ?  
     
   **Using the Remainder Theorem, the remainder is 7.**
6. If the remainder when is divided by is 94, what is the remainder is divided by ?  
     
   **Since the second polynomial is the first polynomial increased by 6, the remainder using the same factor will be the first remainder plus 6, or .**
7. Zoe and Jose tried to figure out if is a factor of . Zoe did it by dividing, and Jose did it more quickly with the remainder theorem. How did Jose do it?  
     
   **By substituting 7 into the polynomial:  
     
   Yes. is a factor of   
   .**
8. If , then What is one factor of ?  
     
   **Using the Remainder Theorem, the factor for would be , or .**

## 1.4 Polynomial Equations

A *polynomial equation* like involves an equal sign with a polynomial expression on one or both sides. The solution set of a polynomial equation is the set of numbers that make the left side of the equation equal to the right side of the equation. Polynomial equations usually have more than one solution.

**Solving Quadratic Equations that Have No x-Term**

A *quadratic equation* is one where the largest exponent is a 2. The simplest type of quadratic equation is when there is no x-term, such as the quadratic equation .

**Solving Factored Polynomial Equations**

The equation is a quadratic equation. If the left side was simplified, the highest exponent would be 2.

**Solving Quadratic Equations by Factoring**

Not all quadratic polynomials factor. If one does an equation where there is a zero on the right-hand side of the equal sign, the solution set can be found very quickly.

**Solving Quadratic Equations with the Quadratic Formula**

When the quadratic expression does not factor, the equation has irrational roots and can be solved with the quadratic formula.

The two solutions to the quadratic equation:

### Check Your Understanding of Section 1.4

1. Multiple-Choice
2. What is the solution set of ?  
   **(2) {4, -4}**
3. What is the solution set of   
   ?  
   **(1) {4, 7}**
4. What is the solution set of   
   ?  
   **(4) {3, -9}**
5. What is the solution set of   
   ?  
   **(4)**
6. What is the solution set of   
   ?  
   **(1)**
7. What is the solution set of ?  
   **(1)**
8. Which equation has the solutions   
   ?  
   **(3)**
9. Which equation has the solutions  
   ?  
   **(4)**
10. What are the solutions to ?

**(3)**

1. What are the solutions to ?  
   **(2)**
2. *Show how you arrived at your answers*.
3. Lila solves the equation by first factoring into . Skylar solves the same equation by first adding 25 to both sides of the equation to get . Who is right?  
     
   **Both Lila and Skylar are right, as both approaches leads to the solution set of   
   {5, -5}.**
4. What are the three solutions to  
    ?
5. Create an equation that has the three solutions and 5.
6. In terms of , what is the solution to the quadratic equation ?
7. Noelle used the quadratic formula to solve . Delilah solved it without the quadratic formula. What did Delilah notice that enabled her to solve this equation without using the quadratic formula?  
     
   Delilah noticed that the quadratic formula could be factored, and possible factors included (1,16), (2,8) and (4,4). The factor (2,8) totals 10 and the product of 2 and 8 is 16.

## 1.5 Quadratic Graphs

A graph of the solution set of a two-variable quadratic equation, like , has a shape called a *parabola*. To produce an accurate sketch of this graph, determine the *vertex* of the parabola and the *x-intercepts* of the parabola if any exist. Depending on the form of the equation, you can use several methods to produce this graph.

The most basic quadratic graph is the graph of   
 Ordered pairs include: (0,0), (1,1), (2,4),(3,9), (-1,1), (-2,4), and (-3,9).

The lowest point, which is (0,0) for this parabola, is the parabola’s *vertex*.

A graph of a function

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This graph is also the graph of the quadratic function

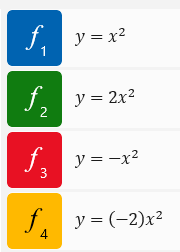
**Changing the *a* Coefficent**

If the coefficient greater than 1 is in front of the , such as , the graph is still a parabola. However, it is narrower than the one created by   
. The coefficient is usually called , so the equation is . The vertex is still at (0,0).

A graph of a line

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If *a* is negative, the parabola will be reflected over the *x-axis* and resemble an upside-down “U”.



A graph of a function

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**Vertex Form of a Quadratic Equation**

A quadratic equation like is said to be in *vertex form*.

Vertex form is an expression like , where a, h, and k are replaced with numbers. When a quadratic equation is in vertex form, the x-coordinate of the vertex is the opposite of the constant in parentheses and the y-coordinate of the vertex is the constant outside the parentheses.

For , the vertex is (-(-2), -5) = (2, -5).

**Math Facts**

The vertex of the parabola defined by   
 is If *a* is greater than 1, the parabola will be narrow. If *a* is between 0 and 1, the parabola will be wide. If *a* is negative, the parabola will resemble an upside down “U”.

**Graphing Quadratic Equations**

To sketch a more accurate graph, find the x-intercepts and y-intercepts of the parabola. To find the *x-intercepts*, set and solve for the values (if there are any) of . The *y-intercept* can be found by setting and solving for y.

Vertex form is related to graphs of transformed functions. The equation function   
 is related to the basic quadratic equation The function can be expressed as . The graph of a function defined this way is a translation of the original graph 2 units to the right and 5 units down. When the vertex of the graph of , which is (0,0), is translate 2 units to the right and 5 units down, the vertex of the transformed parabola will be (2,-5).

**Standard Form of a Quadratic Equation**

Most quadratic equations and two-variable quadratic equations are not written in vertex form. Instead, they look like or . An equation in this simplified form is standard form, . When the equation nis in standard form, determining the vertex takes more work.

**Math Facts**

The x-coordinate of the vertex of the graph of   
 is . The y-coordinate of the vertex is the value of . The vertex of the parabola defined by , has an x-coordinate of and a y-intercept of .

**Graphing Quadratic Equations on the Graphing Calculator**

**Focus and Directrix of a Parabola**

Every parabola has an invisible line and invisible point associated with it. The invisible line is called the *directrix*, and the invisible point is called the *focus*.

For a parabola that resembles a right-side-up “U,” the focus is a bit above the vertex. The directrix is a horizontal line below the vertex. The distance between the focus and vertex is the same as the distance between the vertex and the directrix.

For a parabola that resembles an upside-down “U,” the focus is a bit below the vertex and the directrix is a bit above. Again the distance between the focus and vertex is the same as the distance between the vertex and the directrix.

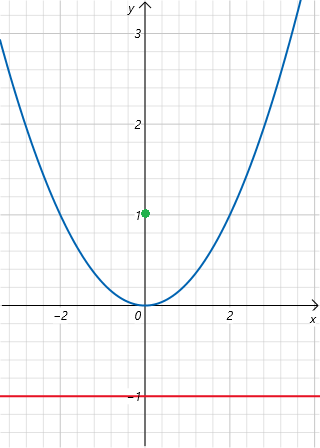
**Math Facts**

The distance between the vertex and the focus is the same as the distance between the vertex and the directrix. If the parabola is defined as   
 or as , that distance is equal to and is usually denoted by the variable p.

For the parabola , the value of p is   
. So the focus is a point located 1 unit above the vertex, at (0, 1). The directrix is a horizontal line located 1 unit below the vertex, at .

A screenshot of a cell phone

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**Math Facts**

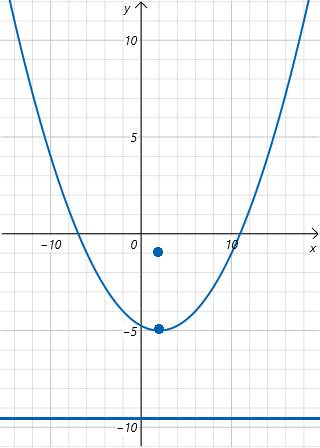
If the vertex of a parabola is at (h, k), the focus is at (h, k + p), and the directrix is at y = k – p, where   
.

**Example 6**

What are the coordinates of the focus and the equation of the directrix, of the parabola defined by   
.

*Solution*: Since this equation is in vertex form, the vertex is .

Since , . The focus is at   
, and the directrix is at   
.



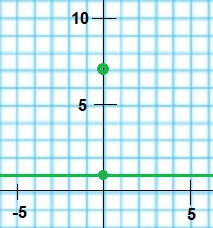
If the focus and directrix are known, it is possible to find the equation of the parabola related to that focus and directrix. The vertex is the midpoint of the vertical line segment connecting the focus to the directrix.

The distance p is half the length of that vertical line segment, and the a-value for the quadratic equation equals .

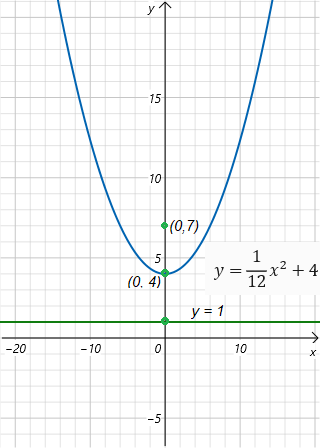
**Example 7**

What is the equation in standard form of a parabola that has its focus at (0, 7) and a directrix of ?

*Solution*: First sketch the focus and directrix on graph paper. Then draw the vertical segment joining the focus and direction.



The value of p is half the length of the segment, and the vertex is the midpoint of the segment. The segment is length 6 (7-1), so p is 3 and   
. The midpoint of the segment is at (0, 4). In vertex form, the equation is   
. In standard form, it is  
.



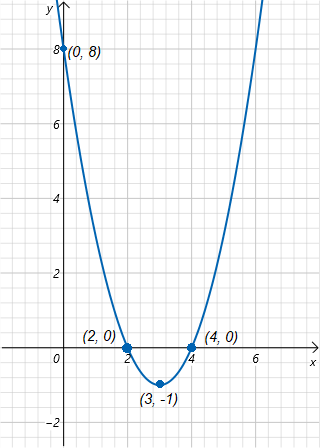
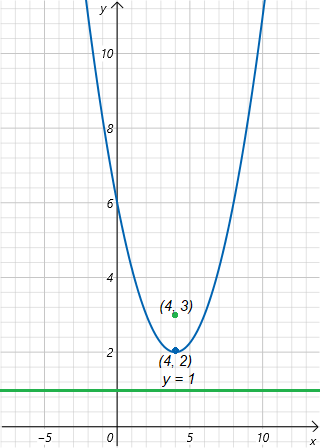
**Math Facts**

For any point on a parabola, the distance from that point to the parabola’s focus is the same as the distance from that point to the parabola’s directrix.

### Check Your Understanding of Section 1.5

1. Multiple-Choice
2. Which is a point on the graph of ?  
   **(4) (3, 18)**
3. What is the vertex of the parabola defined by ?  
   , Vertex:   
   **(1) (-4, -5)**
4. What is an equation of the parabola with its vertex at (3, -1)?  
   , Vertex:   
   **(2)**
5. What are the x-intercepts of the parabola whose equation is ?  
   y = 0,   
   **(1) (4, 0), (-2, 0)**
6. What is the vertex of the parabola whose equation is   
      
   Vertex: (**(1) (-2, -16)**
7. What are the x-intercepts of the parabola whose equation is ?  
      
   **(1) (6, 0), (-2, 0)**
8. Which is the graph of ?  
   Factor:   
      
   x-intercepts:   
   Vertex: , Vertex: (-1, -4)  
   **(2)**
9. What is the equation of the directrix and the coordinates of the focus of the parabola   
   ?  
      
   Vertex: ,   
   Vertex:   
      
   **(3) and ,**
10. What is the equation of the parabola that has the focus and directrix on this coordinate plane?  
    Focus: (0, 4)  
    Directrix:   
       
    Vertex: (0, 0)   
    **Google agrees with me, not with the book.**
11. Which is the graph of ?  
    Vertex form:   
    Vertex: (-3, 5)  
       
    If *a* is negative, the parabola will be reflected over the *x-axis* and resemble an upside-down “U”.

**(1)**

1. *Show how you arrived at your answers*.
2. Sketch the graph of , including the vertex, x-intercepts and y-intercept.  
      
   y-intercept: : (0, 8)  
   x-intercepts:   
   x-intercepts: (2, 0), (4, 0)  
   Vertex:   
   Vertex: (3, -1)  
     
   
3. has a vertex at (-4, -9). What is the value of ?  
   Vertex:
4. Convert into vertex form.  
   Vertex:   
   **Vertex: (5, -2) *(h, k)***
5. What is the equation of the parabola with its focus at (4, 3) and its directrix at ?  
     
   The focus is .   
     
   The Directrix is:   
     
   The distance p is half the length of that vertical line segment, and the a-value for the quadratic equation equals .  
   Directrix:   
     
   Vertex: 
6. If = 8, what is the length of ?   
     
   Any point on the parabola is equidistant from the focus and the directrix.

## 1.6 Polynomial Graphs

**Key Ideas**

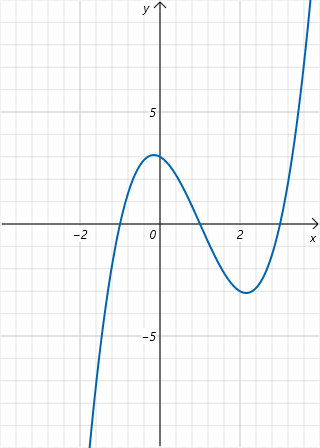
Graphs of cubic and other higher-degree polynomials can be sketched by knowing a few key points on the graph and having an understanding the general shape of curves based on higher-degree polynomials. The *x-intercepts* and *y-intercept* are important as well as having an understanding of the *end behavior* at both ends of the curve.

**Graphing Basic Cubic Equations**

The graph of a quadratic function resembles a “U” (whether it is right-side up or upside down). However the graph of a cubic (3rd-degree) equation resembles a capital “N”. The “N” can be either right-side up or upside down.

A black numbers and a white background

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It is useful if the cubic function you want to graph is already in factored form, such as

If you were to multiply this out,, the coefficient of the -term would be +1, so the a-value is positive. The graph resembles a right-side-up “N”.

This equation can be solved by finding the x-values that would make any of the factors equal to zero.

So the x-intercepts are (-1, 0), (1, 0), and (3, 0).

The y-intercept can be found by substituting into the equation and solving for y.

So the y-intercept is (0, 3).

Plot the four points, and join them with a curve like a right-side-up “N.”

**Graphing Polynomial Functions That Have Double Roots**

**Example 1**

A factored polynomial like

has roots at 2, 2, and -3. Since there are two 2s, the graph of this function has only two intercepts.

**Math Facts**

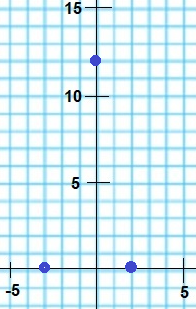
At a double root, the graph of the function “bounces off” the x-axis!

The x-intercepts are (-3, 0) and (2, 0).

The y-coordinate of the y-intercept is at

So the y-intercept is (0, 12).

Since the graph should look like a right-side-up “N,” start from the bottom left of the paper and draw toward the first x-intercept, (-3, 0).



A graph of a function

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**Example 3**

If a, b, and c are all positive integers, which would be the graph of ?

Factoring:

x-intercepts: (b, 0), (-c, 0), (c, 0)

Since a is positive, the graph will resemble a right-side-up “N.”

If we make up 3 positive integers for a, b, c, like .

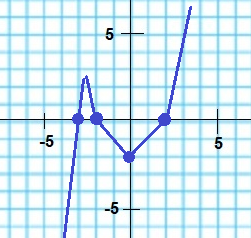
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A graph of a function

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### Check Your Understanding of Section 1.6

1. Multiple-Choice
2. Which could be the equation for this graph?  
   x-intercepts: (1, 0), (3, 0), (6, 0)  
   y-intercept: (0, -2)  
   **(1)**
3. Which could be the equation for this graph?  
   y-intercept: (0, -2)  
   x-intercepts: (2, 0), (2, 0) (4, 0)  
   **(1)**
4. What are the xintercepts and y-intercept of   
   ?  
   **(3) (2, 0), (-4, 0), (-5, 0), (0, -40)**
5. If *a* an *b* are both positive numbers, which could be a graph of ?  
   x-intercepts: (-a, 0), (-a, 0), (b, 0)  
   y-intercept:   
   **(3)**
6. Which equation could this be the graph of?  
   **(3)   
   Books answer of (4) is incorrect because y would always be positive, and the graph shows negative y values below x = 2.**
7. If this is the graph of ,   
   what is the value of a?  
   a must be negative because of upside-down “N”.  
   y-intercept: (0, -2)  
   **(4)**
8. If a and b are positive integers, which could the graph of ?  
   Factor:   
   x-intercepts: (b, 0), (-a, 0), (-a, 0)  
   y-intercept:   
     
   **(1) double-root at (-a, 0), y-intercept is negative**
9. What are the x-intercepts of   
   ?  
   Factor: x-intercepts: (2, 0), (3, 0), (-3, 0)  
   y-intercept (x=0): 18  
   **(2) (2, 0), (3, 0), (-3, 0)**
10. What are the x-intercepts of the graph of   
    ?  
    Factor:   
    (0, 0), (-4, 0), (2, 0)  
    **(4) (0, 0), (2, 0), (-4, 0)**
11. What is the least number of x-intercepts an equation of the form can have?  
    For large negative values of x, will be a much larger negative value than is positive, making it a large negative value.  
    For large positive values of x, will be a much larger positive value than is positive, making it a large positive value.  
      
    There will be at least one x-intercept where y passes from a negative to a positive value  
      
    **(2) 1**
12. *Show how you arrived at your answers*.
13. Sketch the graph of   
    .   
    Include all x- and y-intercepts.  
    **x-intercepts: (2, 0), (-2, 0), (-3, 0)  
    y-intercept (x=0): (0, -2)**  
      
      
    A black text on a white background

    AI-generated content may be incorrect.  
    A graph of a function

    AI-generated content may be incorrect.
14. What is the equation of this graph?  
    x-intercepts: (-1, 0), (3, 0), (3, 0)  
    y-intercept: (0, 3)

A math problem with black lines

AI-generated content may be incorrect.A graph of a function

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1. A graph of a cubic equation has a double root at and a single root at . What could be the equation?  
     
   **where a is any real number**
2. What is the main difference between the equations that represent these two graphs?  
     
   **The coefficient (a-value) of the term is positive for the first graph and negative for the second graph.**
3. One of the x-intercepts of the cubic equation   
    is (2, 0). What are the other two intercepts?  
   One factor:   
   Factor: **Other two intercepts: (-5, 0), (-1, 0)**