**Barron’s Let’s Review Regents – Algebra II**

# Chapter 1: Polynomial Expressions and Equations

## Polynomial Arithmetic

**Multiplying a Polynomial by a Constant**

**Adding Polynomials**

1. Combine the two terms.
2. Combine the two terms.
3. Combine the two constant terms.

**Subtracting Polynomials**

The minus sign can be thought of as multiplying by a negative one. Distribute the multiplication through the second polynomial.

**Multiplying Polynomials**

**FOIL**

1. **F**irst terms in each expression.
2. **O**uter terms in each expression.
3. **I**nner terms in each expression.
4. **L**ast terms in each expression.

**Multiplication Patterns**

**Perfect Squares**

The coefficient of the -term will be twice the constant term.

**Difference of Perfect Squares**

**Dividing Polynomials**

Answer:

### Check Your Understanding of Section 1.1

1. Multiple-Choice
2. What is   
   **(1)**
3. What is ?  
   **(2)**
4. What is ?  
   **(4)**
5. What is ?  
   **(1)**
6. What is ?  
   **(4)**
7. What is ?  
   **(4)**
8. What is ?  
   **(1)**
9. What is ?  
   **(2)**
10. What is ?  
    **(1)**
11. What is ?  
    **(4)**
12. What is ?  
    **(3)**
13. *Show how you arrived at your answers*.
14. Simplify .
15. Zahra calculated   
     as  
     and got   
     What mistake did Zahra make?  
      
    **Zahra made the mistake of not distributing the minus sign in the second expression.**  
    **It should have been:**
16. If , what is the value of *a*?  
      
    **)** (  
     (ck)
17. If , what is the value of *a*?
18. Simplify .

## 1.2 Polynomial Factoring

*Factoring* an integer means finding two other integers (other than 1) whose product is equal to the original integer. The integer 15 has two factors: 3 and 5.

When a polynomial is factored, the factors can provide useful information about the polynomial that was not apparent in the non-factored form.

Just like some numbers can’t be factored (for example, 7, and other prime numbers), some polynomials cannot be factored either. When a polynomial can be factored, there are several different methods of obtaining the factorization, depending on the polynomial.

**Greatest Common Factor Factoring**

*Greatest common factor* factoring is the first type of factoring you should always try. If all the terms of a polynomial have a common factor, that common factor can be “factored out.” Often, the only common factor is 1, in which case this type of factoring is not useful.

**Factoring a Quadratic Trinomial into the Product of Two Binomials**

What is the opposite of FOIL? The opposite is factoring a quadratic trinomial like into the product of two binomials.

If the trinomial is of the form , find two numbers that have a sum of b and a product of c.

Perfect Square Trinomial Factoring

The trinomials , and   
, are three examples of perfect square trinomials. These can be factored into , , and , respectively.

The way to recognize a perfect square trinomial of the form is to compare *c* to .

If , the trinomial is a perfect square and can be factored as

**Difference of Perfect Squares Factoring**

A quadratic expression like is known as the difference between two perfect squares since each of the terms is a perfect square and there is a subtraction sign between the two terms.

**Factoring Cubic Expressions by Grouping**

Polynomial expressions that have one of the variables raised to the third power are called cubic polynomials. Generally, they are very difficult to factor. Sometimes a technique called factor by grouping can be used to factor certain cubic polynomials.

Factor by grouping when you have a four-term cubic expression and you factor a common factor from the first two terms and another common factor from the last two terms. Then you cross your fingers and hope there will be a new common factor that you can then factor out.

For example, the polynomial can be factored this way.

An can be factored out of the first two terms, and a 4 can be factored out of the last two terms.

At this point, notice that both of the terms have a factor of . This can be factored out to become  
.

**Factoring More Complicated Expressions**

Polynomials that have exponents greater than 3 can sometimes be factored by rewriting the polynomials in an equivalent form that can be factored with other methods.

The expression has an exponent of 6. Since 6 is an even number, can be expressed as . Since 16 is a perfect square, the original expression can be written as , which can then be factored with the difference of perfect squares pattern: .

**Algebra Identities**

When two algebraic expressions can be simplified to the same expression, it is called an identity. Proving that something is an identity requires simplifying one or both sides of the equation until the two sides are identical. Until an identity is proved, there will be a small question mark over the equal sign like ≟. After the identity is established, the question mark over the equals sign is replaced with a .

**Number Theory Proofs**

Sometimes a theorem about numbers can be proved by turning the theorem into an identity to be verified.

### Check Your Understanding of Section 1.2

1. Multiple-Choice
2. Which shows factored?  
   **(1)**
3. Which shows factored?  
   **(1)**
4. Which shows factored?  
   **(3)**
5. Which shows factored?  
   **(4)**
6. Which shows factored?  
   **(4)**
7. Which shows factored?  
   **(2)**
8. Which shows factored?  
   **(2)**
9. Which shows factored?  
   **(1)**
10. Which shows factored?  
    **(1)**
11. Which shows ?  
    **(3)**
12. Show how you arrived at your answers.
13. Factor .  
    Factors: 1, 3, 2x, x
14. Factor .  
    Factors: 17, 39
15. What value for *c* can be factored into ?  
    *c* = 81
16. How can the fact that be used to find the factors (not including (1 or 551) of 551?  
      
    **Difference of perfect squares**
17. Completely factor .  
    Factors: (1,36), (2,18), (3,12), (4,9), (6,6)

## 1.3 The Remainder Theorem and the Factor Theorem

When something is a factor of a number, like 5 is a factor of 10, there will be no remainder when the number is divided by the factor. This is called *the factor theorem*. When something is not a factor of a number, like 3 is not a factor of 10, there will be some remainder when the number is divided by the factor. With polynomial division, there is a theorem called *the remainder theorem* that enables you to determine the remainder of some divisions without going through the long division process.

**The Remainder Theorem**

If you divide the polynomial function   
 by with the long division process, you get remainder 5.The remainder theorem says that you will also get the number 5 if you substitute +4 (the opposite of -4 in the divisor) into the   
.

To check if the remainder theorem works for this example, evaluate

**Math Facts**

The remainder theorem says that the remainder when a polynomial equation is divided by is equivalent to the value of the polynomial when is substituted for . If the expression is , then substitute into the polynomial.

**The Factor Theorem**

When a binomial like is a factor of a polynomial like it means there will be remainder of zero when is divided by If you evaluate the polynomial   
 for , it becomes   
, just as the remainder theorem predicted. From this, we get the factor theorem.

**Math Facts**

The factor theorem says that if is a factor of a polynomial, then the value of the polynomial when is substituted for will be 0.

### Check Your Understanding of Section 1.3

1. Multiple-Choice
2. What is the remainder when is divided by ?  
   (3\*3\*3)+(3\*3)-(9\*3)+5 = 27+9-27+5 = 14
3. What is the remainder when   
    is divided by ?  
   **(2) 3**
4. If , what is the remainder when is divided by   
   **(2)**
5. If the remainder when is divided by is 7, what is the value of a?  
   **(4) 5**
6. If the remainder when is divided by is 97, what is the value of ?  
   **(3) 9**
7. If , what is the remainder when is divided by ?  
   **(4) 0**
8. Which of the following is a factor of   
   ?  
   1 => Remainder -30  
   2 => 8 + 12 – 20 – 24 = -24  
   3 => 27 + 27 – 30 -24 = 54 – 54 = 0  
   **(3)**
9. Which of the following is a factor of   
   ?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x |  |  |  |  |  | Total |
| 3 | 162 | -243 | -81 | 138 | 24 | 0 |
| -4 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 1 | 2 | -9 | -9 | 46 | 24 | 54 |

**(1)**

1. If is a factor of , what is the value of ?  
   **(4) -60**
2. If is a factor of , what is the value of ?  
   **(3) 6**
3. *Show how you arrived at your answers*.
4. What is the remainder when   
    is divided by ?  
   f(4)
5. . If , what is the reminder when is divided by ?  
     
   **Using the Remainder Theorem, the remainder is 7.**
6. If the remainder when is divided by is 94, what is the remainder is divided by ?  
     
   **Since the second polynomial is the first polynomial increased by 6, the remainder using the same factor will be the first remainder plus 6, or .**
7. Zoe and Jose tried to figure out if is a factor of . Zoe did it by dividing, and Jose did it more quickly with the remainder theorem. How did Jose do it?  
     
   **By substituting 7 into the polynomial:  
     
   Yes. is a factor of   
   .**
8. If , then What is one factor of ?  
     
   **Using the Remainder Theorem, the factor for would be , or .**

## 1.4 Polynomial Equations

A *polynomial equation* like involves an equal sign with a polynomial expression on one or both sides. The solution set of a polynomial equation is the set of numbers that make the left side of the equation equal to the right side of the equation. Polynomial equations usually have more than one solution.

**Solving Quadratic Equations that Have No x-Term**

A *quadratic equation* is one where the largest exponent is a 2. The simplest type of quadratic equation is when there is no x-term, such as the quadratic equation .

**Solving Factored Polynomial Equations**

The equation is a quadratic equation. If the left side was simplified, the highest exponent would be 2.

**Solving Quadratic Equations by Factoring**

Not all quadratic polynomials factor. If one does an equation where there is a zero on the right-hand side of the equal sign, the solution set can be found very quickly.

**Solving Quadratic Equations with the Quadratic Formula**

When the quadratic expression does not factor, the equation has irrational roots and can be solved with the quadratic formula.

The two solutions to the quadratic equation:

### Check Your Understanding of Section 1.4

1. Multiple-Choice
2. What is the solution set of ?  
   **(2) {4, -4}**
3. What is the solution set of   
   ?  
   **(1) {4, 7}**
4. What is the solution set of   
   ?  
   **(4) {3, -9}**
5. What is the solution set of   
   ?  
   **(4)**
6. What is the solution set of   
   ?  
   **(1)**
7. What is the solution set of ?  
   **(1)**
8. Which equation has the solutions   
   ?  
   **(3)**
9. Which equation has the solutions  
   ?  
   **(4)**
10. What are the solutions to ?

**(3)**

1. What are the solutions to ?  
   **(2)**
2. *Show how you arrived at your answers*.
3. Lila solves the equation by first factoring into . Skylar solves the same equation by first adding 25 to both sides of the equation to get . Who is right?  
     
   **Both Lila and Skylar are right, as both approaches leads to the solution set of   
   {5, -5}.**
4. What are the three solutions to  
    ?
5. Create an equation that has the three solutions and 5.
6. In terms of , what is the solution to the quadratic equation ?
7. Noelle used the quadratic formula to solve . Delilah solved it without the quadratic formula. What did Delilah notice that enabled her to solve this equation without using the quadratic formula?  
     
   Delilah noticed that the quadratic formula could be factored, and possible factors included (1,16), (2,8) and (4,4). The factor (2,8) totals 10 and the product of 2 and 8 is 16.